Optimal Maintenance Strategies for Wind Turbine Systems Under Stochastic Weather Conditions

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***Abstract*—We examine optimal repair strategies for wind turbines operated under stochastic weather conditions. In-situ sensors installed at wind turbines produce useful information about the physical conditions of the system, allowing wind farm operators to make informed decisions. Based on the information from sensors, our research objective is to derive an optimal preventive maintenance policy that minimizes the expected average cost over an infinite horizon. Specifically, we formulate the problem as a partially observed Markov decision process. Several critical factors, such as weather conditions, lengthy lead times, and production losses, which are unique to wind farm operations, are considered. We derive a set of closed-form expressions for the optimal policy, and show that it belongs to the class of monotonic four-region policies. Under special conditions, the optimal policy also belongs to the class of monotonic three-region policies. The structural results of the optimal policy reflect the practical implications of the turbine deterioration process.**

***Index Terms*—Dynamic programming, partially observed Markov decision process, random deterioration, stochastic environment, wind turbine operations and maintenance.**

ACRONYMS

NA	no action
PM	preventive maintenance
OB	observation
CM	corrective maintenance
O&M	operations and maintenance
CBM	condition-based monitoring
POMDP	partially observed Markov decision process
AM4R	At-Most-Four-Region
AM3R	At-Most-Three-Region

NOTATION

π	information state
$\tilde{\pi}$	information state when system is stopped for repairs
$\pi'(\pi)$	information state at next decision period
P	transition matrix
$R(\pi)$	reliability
e_i	unit vector with 1 in the ith position
C_{CM}, C_{PM}, or C_{OB}	cost for CM, PM, or OB
T	lead time
τ	revenue losses per period
W_{PM}, W_{CM}	probabilities that adverse weather conditions, which prohibit respectively PM, or CM, occur in a period
$V_n(\pi)$	minimum expected total cost-to-go with n decision periods left when the current state is π
$NA_n(\pi)$, $PM_n(\pi)$, or $OB_n(\pi)$	expected total cost-to-go when NA, PM, or OB is taken
$CM_n(\pi)$	expected total cost-to-go when CM is taken
g	average cost per period
$b(\pi)$	bias under optimal policy
$b_{NA}(\pi)$, $b_{PM}(\pi)$, or $b_{OB}(\pi)$	bias when NA, PM, or OB is taken

I. INTRODUCTION

IN MANY INDUSTRIES, machines are operated under more or less stationary conditions. However, wind turbines suffer from stochastic loadings. Wind speed varies season by season, and day by day [1], [2]. These stochastic loadings make the degradation process rather complex. In addition, the feasibility of conducting maintenance is constrained by weather. Current maintenance practice for wind farms mainly consists of scheduled maintenance, and corrective maintenance (CM). According to [3], and [4], scheduled maintenance is carried out usually twice a year for a turbine, and there are on average 2.2 failures per turbine per year requiring major repairs. Considering today's trend of large-scale wind farms, and their long distance from operation centers, the cost for these maintenance visits is substantial.

Thanks to the advancement of sensor technology, many turbine manufacturers began to install condition-based monitoring (CBM) equipment, with many sensors within turbines. With these sensor signals, one can presumably estimate the turbine's

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physical condition, and make decisions regarding which maintenance actions to take. Consequently, wind farm operators can reduce the number of unnecessary visits, and avoid unexpected, sometimes catastrophic, failures.

CBM equipment provides abundant information, but it does not solve the uncertainty issue perfectly [5]. Fault diagnosis based on sensor measurements is nontrivial due to the fact that wind turbines operate under non-steady operating conditions. Often, it is not possible to conclude the exact state of a turbine component. Instead, one has to estimate the actual state in a probabilistic sense.

Three stochastic factors need to be considered in modeling wind turbine maintenance. The first factor is weather conditions, which may constrain the feasibility of maintenance actions. For example, under high wind speeds of more than 20 meters per second (m/s), climbing up a turbine is not allowed. Under wind speeds higher than 30 m/s, the site becomes inaccessible [6]. On the other hand, wind farms are inevitably located on windy sites to maximize electricity generation. For this reason, repair actions cannot be carried out often. In a study using a Monte-Carlo simulation, wind turbine availability remains only at 85%–94% in a 100 unit wind farm, situated about 35 kilometers off the Dutch coast [7]. The main reason for this relatively low availability is the farm's poor accessibility, which is on average around 60%. In another study by Bussel [3], the availability of a wind farm was 76%.

The second factor is repairing interruption and delay. Most wind farm-related repairs take several days to several weeks to complete. This relatively long duration increases the likelihood that a repair is interrupted by adverse weather conditions. When the weather becomes adverse, the crew must stop working, and wait until weather conditions become favorable. These delays cause revenue losses because wind turbines can no longer be operated until the repairs are completed. The third factor is long lead time for assembling maintenance crews, and obtaining spare parts, which also significantly affects wind turbine downtime. For example, it can take several weeks for parts, such as a gearbox, to be delivered [8].

Due to the aforementioned uncertainty, and stochastic issues, we believe that a properly timed, well-planned preventive maintenance strategy is pressingly needed in the wind power industry. Taking all of the above issues into consideration, we derive the optimal repair strategies that minimize the average long-run cost for wind turbine maintenance under stochastic conditions. We emphasize the main contributions of this study with the following points:

1) We develop a dynamic optimization model by formulating the problem as a partially observed Markov decision process (POMDP), which considers the costs associated with different actions, and other critical aspects. To the best of our knowledge, the proposed model is the first mathematical model for wind turbine maintenance.
2) We analytically derive the optimal control limits for each action as a set of closed-form expressions. We provide the necessary and sufficient conditions under which preventive maintenance will be optimal. The sufficient conditions for other actions to be optimal are also derived.
3) We establish several structural properties, such as the monotonicity of the optimal policy. We show that the structure of the optimal policy is similar to those studied in the previous POMDP literature, but our policy structure requires *weaker* assumptions. Optimality results for other policy structures, not previously proved in the literature, are also presented. We examine the practical implications of these properties in wind turbine maintenance.

The remainder of the paper is organized as follows. We start off with reviewing related work in Section II. Then, we present the POMDP model in Section III. In Section IV, several structural properties of the optimal policy are discussed. In Section V, we derive an algorithm for finding the optimal policy based on the structural properties established in the previous section. The computational results are reported in Section VI. Finally, we conclude the paper in Section VII.

II. Literature Review

Several studies have been conducted to find critical factors which affect the operations and maintenance (O&M) costs of wind turbines. Pacot *et al.* [8] discuss key performance indicators in wind farm management, and review the effects of several factors such as turbine age, turbine size, and location. Bussel [3] presents an expert system to determine the availability and O&M costs. Rademakers *et al.* [7] describe two simulation models for O&M, and illustrate the features and benefits of their models through a case study of a 100 MW offshore wind farm.

An insightful review of recent CBM work for wind turbines is provided by Caselitz & Giebhardt [9]. The most widely used monitoring system is vibration monitoring. The other monitoring systems include measuring the temperature of bearings, lubrication oil particulate content analysis, and optical strain measurements [10]. Nilsson & Bertling [4] discuss the benefits of CBM with a case study of two wind farms by breaking down the entire maintenance costs into several components. McMillan & Ault [6] also quantify the cost-effectiveness of CBM using a Monte Carlo simulation.

Several mathematical models that incorporate information from CBM sensors have recently been introduced. Although these models are not specifically developed for wind turbine maintenance, they provide insights into how CBM sensory information can be utilized. Maillart [11] uses POMDPs to adaptively schedule observations, and to decide the appropriate maintenance actions based on the state information from CBM sensors. Gebraeel [12] integrates the real-time sensory signals with a population-specific aging process so to capture the degradation behavior of individual components. Similarly, Ghassemi *et al.* [13] represent a system's deterioration process with two sources. One source is the average aging behavior that is usually provided by the manufacturer or estimated using survival data. The other source is the system utilization that can be diagnosed by using CBM data.

Several studies examine the structural properties of POMDP maintenance models [11], [14]–[19]. Although these studies use different state definitions and cost structures, they establish a similar structural property of the optimal policy called the monotonic "At-Most-Four-Region" (AM4R) structure. The monotonic AM4R structure implies that, along ordered subsets of deterioration state spaces, the optimal policy regions are divided into at most four regions with the following order: no action → observation (or, inspection) → no action → preventive maintenance. For detailed reviews of these AM4R studies, we refer the reader to [11].

Few quantitative studies have been done for systems operating under stochastic environments. Thomas *et al.* [20] investigate the repair strategies to maximize the expected survival time until a catastrophic event occurs in an uncertain, stochastic environment. These authors consider the situation where a system should be stopped during inspection or maintenance action. If specific events, called "*initiating events*", take place when a system is down, or being replaced, it is denoted a catastrophic event. They show that similar AM4R structural results hold for a simple system where a system state takes only binary values, i.e., operating or failed.

In this study, we devise a multistate, POMDP model to represent the degradation process of wind turbines, and to decide the optimal maintenance strategies. Our model extends the model introduced in [11] by incorporating several unique characteristics of wind turbine operations. To represent the stochastic weather conditions, our extended model adopts the "*initiating events*" idea, proposed in [20]. This approach is applicable because the occurrence of harsh weather conditions delay repairs, and cause significant revenue losses, making the circumstances analogous to those discussed in [20]. Other aspects of wind turbine operations, such as the long lead time after an unplanned failure, and the resulting production losses, are also included in our model.

III. Mathematical Models

In this section, we formulate the wind turbine maintenance problem, and introduce the existing algorithm to numerically solve it. In later sections, we present a computationally improved algorithm after analysing the structural properties of the proposed model.

A. Model Formulation

We consider a system whose deterioration conditions are classified into a finite number of levels $1, \cdots, m+1$. Status 1 denotes the best condition, "new". Status m denotes the most deteriorated condition, and status $m+1$ is the failed condition. Often, one does not know a system's physical condition precisely. We therefore choose to define a state in a probabilistic sense to represent the belief over the actual deteriorated condition, and formulate the problem using a POMDP model. That is, the state of a system is defined as

$$\pi = [\pi_1, \pi_2, \cdots, \pi_{m+1}], \tag{1}$$

where π_i, $i = 1, \cdots, m+1$ is the probability that the system is in deterioration level i. Please note that this state definition is consistent with what has been used in the POMDP literature [19], where π is commonly known as an information state. Because wind turbines do not properly operate upon failure, we say that $\sum_{i=1}^{m} \pi_i = 1$ when the system is not failed; if it fails, $\pi_{m+1} = 1$.

We assume that wind farm operators make decisions in discrete time. Conceptually, each decision period can be of any length of time. But, we consider that decisions are made frequently (e.g. weekly) because wind farm operators want to make timely decisions based on the stream of sensor signals from CBM equipment. When decisions are made frequently, a discount rate is close to 1. Therefore, we formulate the problem as an average expected cost model as suggested in Puterman [21], and we examine the policies to minimize the expected cost per unit period.

Given the information state π, one of the following *three* actions is available for an operating system at the beginning of each period.

- No Action (NA): continue the operation without any intervention. The system undergoes Markovian deterioration according to a known transition probability matrix, $P = [p_{ij}]_{(m+1)\times(m+1)}$ [22], [23]. Suppose that the current information state is π, and no action is taken. The probability that the system will still operate until the next decision point is $R(\pi) = 1 - \sum_{i=1}^{m} \pi_i p_{i,m+1}$. In the literature, $R(\pi)$ is often referred to as the *reliability* of the system. Maillart [11] shows that the information state after the next transition, given the system is not failed, is

$$\pi'_j(\pi) = \begin{cases} \frac{\sum_{i=1}^{m} \pi_i p_{ij}}{R(\pi)}, & j = 1, 2, \ldots, m \\ 0, & j = m+1 \end{cases} \tag{2}$$

  As such, the system is transited to the next state $\pi'(\pi) = [\pi'_1(\pi), \cdots, \pi'_m(\pi), 0]$ with probability $R(\pi)$. If it fails with probability $1 - R(\pi)$, the state becomes e_{m+1} in the next period.
- Preventive Maintenance (PM): repair the system at cost C_{PM} $(\leq C_{CM})$. PM takes one full period. We assume that, to complete PM, weather conditions should be favorable during one full period. In the case that weather becomes harsh during PM, the crew hold the repair work until the weather becomes favorable. This delay incurs τ revenue losses per period because wind turbines cannot be operated until PM is completed. Let $\tilde{\pi}$ denote the state when the system is stopped for repairs. After PM, the state is returned to an as-good-as-new state.
- Observation (OB): evaluate the exact deterioration level at cost C_{OB} $(C_{OB} + C_{PM} \leq C_{CM})$. OB instantaneously reveals the system state with certainty. So the information state reverts to state e_i, where $e_i = [0, \cdots, 1, \cdots, 0]$ is an $(m+1) \times 1$ dimensional row vector with a 1 in the ith position, and 0 elsewhere. After observation, the decision maker will choose either NA, or PM in that same decision period, based on the updated information state.

Upon a failure, parts are ordered, and a maintenance crew is arranged, which takes T lead time. When all of the parts and crew are available, CM is carried out for one full period at cost C_{CM} if weather conditions are conducive to repair work. Otherwise, CM has to wait until weather conditions become favorable. Unless CM is completed, wind turbines cannot be operated, and it causes τ revenue losses per period. After CM, the

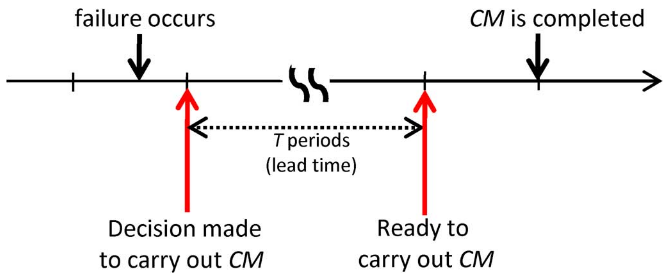

Fig. 1. Corrective maintenance after a failure occurs when maintenance crew and spares are available, and weather conditions are conducive to carry out CM.

system is renewed to an as-good-as-new state, e_1. Fig. 1 illustrates the process after a failure.

When W_{PM} or $W_{CM} > 0$, it implies that the stochastic operating conditions affect maintenance feasibility. On the contrary, $W_{PM} = W_{CM} = 0$ represents a *static* environment in which repair actions can be taken at any time. Because CM requires more complicated repair jobs than PM, CM needs better weather conditions than PM. Therefore, $W_{CM} \geq W_{PM}$ in many practical cases. We assume that the events of these adverse weather conditions occur randomly.

Let $V_n(\pi)$ denote the minimum expected total cost from the current period n to the terminal period (or, cost-to-go) when the current state is π. We formulate the problem as the average expected cost model

$$V_n(\pi)=\min\begin{cases} NA_n(\pi)=(\tau T+CM_{n-T-1}(e_{m+1}))(1-R(\pi)) \\ \qquad +V_{n-1}(\pi'(\pi))R(\pi) \\ PM_n(\pi)=(1-W_{PM})(\tau+C_{PM}+V_{n-1}(e_1)) \\ \qquad +W_{PM}(\tau+PM_{n-1}(\tilde{\pi})) \\ OB_n(\pi)=C_{OB}+\sum_{i=1}^{m} Post_n(e_i)\pi_i \end{cases} \tag{3}$$

where

$$CM_{n-T-1}(e_{m+1})=(1-W_{CM})\left(\tau+C_{CM}+V_{n-T-2}(e_1)\right) + W_{CM}\left(\tau+CM_{n-T-2}(e_{m+1})\right), \tag{4}$$

$$PM_{n-1}(\tilde{\pi})=(1-W_{PM})\left(\tau+C_{PM}+V_{n-2}(e_1)\right) + W_{PM}\left(\tau+PM_{n-2}(\tilde{\pi})\right), \tag{5}$$

$$Post_n(e_i)=\min\left\{NA_n(e_i), PM_n(e_i)\right\} \tag{6}$$

In $NA_n(\pi)$, the first term τT reflects revenue losses during lead time after a failure. $OB_n(\pi)$, and $Post_n(\pi)$ together represent that, after each observation at cost C_{OB}, the state is updated to e_i with probability π_i, and then we choose either NA or PM in the same decision period. $CM_{n-T-1}(e_{m+1})$, and $PM_{n-1}(\tilde{\pi})$ consider weather constraints when carrying out CM, or PM, respectively.

Because the system is renewed after CM or PM as long as weather conditions are good, the model is unichain[1] for $0 \leq W_{CM} < 1$, and $0 \leq W_{PM} < 1$ [11]. For these kinds of problems, Puterman [21] shows that $V_n(\pi)$ approaches a line with slope g, and intercept $b(\pi)$, as n becomes large. That is,

$$\lim_{n\to\infty}\frac{V_n(\pi)}{n}=g, \text{ and } \lim_{n\to\infty}\left(V_n(\pi)-ng\right)=b(\pi) \tag{7}$$

[1]That is, the transition matrix corresponding to each action consists of a single recurrent class.

Here, g denotes the average cost per unit time under the optimal policy, and $b(\pi)$ is the bias, or the relative cost when the information state starts from π.

Taking the limits of both sides of (3), and then applying (7) in both sides, yields

$$b(\pi)=min\begin{cases} b_{NA}(\pi)=\left((\tau-g)T+b(e_{m+1})\right)(1-R(\pi)) \\ \qquad +b\left(\pi'(\pi)\right)R(\pi)-g, \\ b_{PM}(\pi)=(1-W_{PM})(\tau+C_{PM}) \\ \qquad +W_{PM}\left(b(\tilde{\pi})+\tau\right)-g, \\ b_{OB}(\pi)=C_{OB}+\sum_{i=1}^{m} b(e_i)\pi_i \end{cases} \tag{8}$$

Applying the same technique to (4), and (5), respectively, yields

$$b(e_{m+1})=C_{CM}+b(e_1)+\frac{\tau-g}{1-W_{CM}} \tag{9}$$

$$b(\tilde{\pi})=C_{PM}+b(e_1)+\frac{\tau-g}{1-W_{PM}} \tag{10}$$

Because $b(\pi)$ is the relative difference in total cost that results from starting the process in state π instead of in any other state, Puterman [21] suggests to set $b(\pi^0)=0$ for an arbitrary π^0. Intuitively, we set $b(e_1)=0$ in (9), and (10). Let us now define the new maintenance costs which compound weather effects, lead time, and production losses by C'_{CM}, and C'_{PM}, respectively, as

$$C'_{CM}=C_{CM}+\frac{\tau-g}{1-W_{CM}}+(\tau-g)T \tag{11}$$

$$C'_{PM}=C_{PM}+\frac{\tau-g}{1-W_{PM}} \tag{12}$$

Note that both C'_{CM}, and C'_{PM} are increasing in W_{CM}, and W_{PM}, respectively. This increase implies that a higher frequency of harsh weather conditions incurs higher repair costs. Here, we assume that $\tau \geq g$; that is, the revenue losses are greater than or equal to the average cost per period. Therefore, the added costs due to an unplanned failure (that is, $C'_{CM}-C'_{PM}$) arise from the following three factors: increased repair costs (for doing CM), increased possibility of repair delays due to more restricted weather requirements to carry out CM, and production losses caused by the waiting time to prepare resources after a failure.

Substituting C'_{CM}, and C'_{PM} into (8)–(9) simplifies these equations to

$$b(\pi)=min\begin{cases} b_{NA}(\pi)=C'_{CM}(1-R(\pi))+b\left(\pi'(\pi)\right)R(\pi)-g, \\ b_{PM}(\pi)=C'_{PM}, \\ b_{OB}(\pi)=C_{OB}+\sum_{i=1}^{m} b(e_i)\pi_i \end{cases} \tag{13}$$

B. Solution Method—Pure Recursive Technique

First, let us consider a sample path emanating from an information state π. By a sample path, we mean the sequence of information states over time when no action is taken, which is denoted by $\{\pi, \pi^2, \cdots, \Pi(\pi)\}$ where $\pi^2=\pi'(\pi), \pi^3=\pi'(\pi^2)$, and so on. $\Pi(\pi)$, defined by $\Pi(\pi)\equiv\pi^{k^*}$, where $k^*=\min\{k : \|\pi^{k+1}-\pi^k\|<\epsilon\}$ with small $\epsilon>0$, is a stationary state, or an absorbing state. Maillart [11] shows, by referring to [24], that when the Markov chain is acyclic, $\Pi(\pi)$ exists for any $\epsilon>0$.

Let us call the sequence of states emanating from one of the extreme points $b(e_i), \forall i$, in (13) an *extreme* sample path. All the biases at the states on the extreme sample paths and average cost

g can be obtained by applying *policy iteration* (or *value iteration*) methods to the states only on the extreme sample paths [21]. Then, $b_{OB}(\pi)$, and $b_{PM}(\pi)$ in (13) can be directly computed.

Now, we only need to compute $b_{NA}(\pi)$ to get $b(\pi)$. Maillart [25] introduces the following recursive technique. First, we solve (13) for $\Pi(\pi)$ by

$$b(\Pi(\pi)) = min \begin{cases} b_{NA}(\Pi(\pi)) = C'_{CM} - \frac{g}{1-R(\Pi(\pi))}, \\ b_{PM}(\Pi(\pi)) = C'_{PM}, \\ b_{OB}(\Pi(\pi)) = C_{OB} + \sum_{i=1}^{m} b(e_i)\Pi(\pi)_i \end{cases} \quad (14)$$

Then, we apply $b(\Pi(\pi))$ to (13) to find the optimal policy at the previous state. By solving the recursive set of equations backwards, we can get the optimal policy along the states on the sample path emanating from the original state π.

However, this recursive technique might be computationally inefficient when we want to find the optimal policies at a large number of states in a high dimensional state space. This inefficiency is because we have to apply each recursive set of equations for each state. These computational difficulties motivate us to study the structural properties of the model.

IV. Structural Properties

In this section, we establish several structural properties of the optimal policy. More specifically, we derive a set of closed expressions for the optimal policy including the exact control limits for PM. In later sections, we show how these results help attain optimal polices. We also show that the model exhibits the monotonous AM4R policy structure. This finding is an extension of a previous study in [11]. In [11], the AM4R results are shown, for a simpler model than the one presented here, and are obtained under specific assumptions on the transition matrix, and information states. We relax the assumptions while proving the results, and establish the conditions when the optimal policy is simplified to a more intuitive "At-Most-Three-Region" (AM3R) structure.

A. Preliminary Results

We first introduce several definitions which are often used in POMDP studies. These definitions can be found, for example, in [15], [16], and [18].

Definition 1: Information state π is stochastically less (or smaller) than $\hat{\pi}$, denoted as $\pi \prec_{st} \hat{\pi}$ iff $\sum_{i=j}^{m+1} \pi_i \leq \sum_{i=j}^{m+1} \hat{\pi}_i$ for all $j = 1, \cdots, m+1$.

Definition 2: Information state π is less (or smaller) in likelihood ratio than $\hat{\pi}$, denoted as $\pi \prec_{lr} \hat{\pi}$ iff $\pi_i \hat{\pi}_j - \pi_j \hat{\pi}_i \geq 0$ for all $j \geq i$.

These two definitions present the binary relations of the two states in the sense of deterioration. Both definitions imply that, when the system is less deteriorated, the state is stochastically (or in the likelihood ratio) less than another [19]. However, Proposition 1(a) (see below) suggests that the $\prec_{lr}$ relationship is stronger than the $\prec_{st}$ relationship [16]. We also need additional definitions regarding the transition matrix P.

Definition 3: A transition matrix P has an Increasing Failure Rate (IFR) if $\sum_{j \geq k} p_{ij} \leq \sum_{j \geq k} p_{i'j}$ for all $i' \geq i$, and $\forall k$.

Definition 4: A transition matrix P is Totally Positive of order 2 (TP2) if $p_{ij} p_{i'j'} \geq p_{i'j} p_{ij'}$ for all $i' \geq i$, and $j' \geq j$.

These definitions imply that the more deteriorated system tends to more likely deteriorate further, and/or fail [11]. Similar to the stochastic relations defined in Definition 1, and Definition 2, $TP2$ is a more stringent assumption than IFR due to the following Proposition 1(b) [16].

Proposition 1: (Rosenfield [16]) (a) If $\pi \prec_{lr} \hat{\pi}$, then $\pi \prec_{st} \hat{\pi}$. (b) If P is $TP2$, then P is IFR.

Before presenting our results, we introduce several well-known results in the following two Propositions.

Proposition 2: (Derman [26]) For any column vector v such that $v_i \leq v_{i+1}, \forall i$, if $\pi \prec_{st} \hat{\pi}$, then $\pi \cdot v \leq \hat{\pi} v$.

Proposition 3: (a) (Maillart [11]) Suppose that P is IFR. If $\pi \prec_{st} \hat{\pi}$, then $R(\pi) \geq R(\hat{\pi})$. (b) (Maillart & Zheltova [19]) If P is IFR and $\pi \prec_{st} \hat{\pi}$, then $\pi P \prec_{st} \hat{\pi} P$.

Now, Proposition 4 establishes that, when P is IFR, the stochastic ordering of two states are maintained after the transitions. Proofs of all propositions, lemmas, and theorems are included in the Appendix, or on the author's website [34].

Proposition 4: Suppose that P is IFR. If $\pi \prec_{st} \hat{\pi}, \pi'(\pi) \prec_{st} \pi'(\hat{\pi})$.

The following Proposition 5 demonstrates that the optimal cost-to-go for a failed system is always greater than, or equal to, the cost-to-go when it is stopped for PM.

Proposition 5: (a) $CM_n(e_{m+1}) - C_{CM} \geq PM_n(\pi) - C_{PM} \forall n$ where $CM_n(e_{m+1})$, and $PM_n(\pi)$ are defined in (4), and (3), respectively. (b) $CM_n(e_{m+1}) \geq PM_n(\pi) \forall n$.

The above Propositions allow us to derive the monotonicity of $V_n(\pi)$ in $\prec_{st}$-ordering, as shown in Lemma 1.

Lemma 1: If P is IFR, $b(\pi)$ in (13) is non-decreasing in $\prec_{st}$.

The claim of Lemma 1 extends the result presented in [11] where the monotonicity of the optimal cost function in $\prec_{lr}$-ordering on the $TP2$ transition matrix is shown. Also, unlike our model, the model in [11] assumes $\tau = 0, T = 0$, and static environments (that is, $W_{CM} = W_{PM} = 0$). Therefore, the result of Lemma 1 is more general, and can be applied to other general aging systems.

B. Closed Expressions for Optimal Policy Regions

In this section, we present the closed boundary expressions for the optimal policy. Let $\delta^*(\pi)$ denote the stationary optimal policy (or decision rule) at π. Also, let $\Omega_{NA}(\pi)$, $\Omega_{OB}(\pi)$, $\Omega_{PM}(\pi)$ be the set of information states with $\delta^*(\pi) = NA$, $\delta^*(\pi) = OB$, and $\delta^*(\pi) = PM$, respectively. To get the optimal policy to minimize the long-run average cost, we need to compare $b_{NA}(\pi)$, $b_{PM}(\pi)$, and $b_{OB}(\pi)$.

First, the following Lemma 2 explains when NA is preferred to PM, and vice versa. To prove the claim, we apply a technique similar to the one used in [13].

Lemma 2: Suppose that P is IFR and upper-triangular. $\delta^*(\pi) \neq PM$ if $R(\pi) \geq 1 - g/(C'_{CM} - C'_{PM})$. Also, $\delta^*(\pi) \neq NA$ if $R(\pi) < 1 - g/(C'_{CM} - C'_{PM})$ for $\pi \prec_{st} \pi'(\pi)$.

The claim of Lemma 2 is intuitive. As the system deteriorates, its reliability monotonically decreases. When its reliability is lower than a threshold (here, it is $1 - g/(C'_{CM} - C'_{PM})$), it is better to take some actions rather than do nothing. On the contrary, we do not need to carry out costly maintenance action for a highly reliable system. Note that the second part of Lemma 2 requires the assumption $\pi \prec_{st} \pi'(\pi)$, which implies that the next state is more deteriorated than the current state in a probabilistic sense. This assumption should hold in most commonly encountered aging systems.

With the result of Lemma 2, $b_{OB}(\pi)$ in (13) can be reformulated as

$$b_{OB}(\pi) = C_{OB} + \sum_{i=1}^{m} \{b_{NA}(e_i) \cdot I(R(e_i) \geq \alpha) + b_{PM}(e_i) \cdot I(R(e_i) < \alpha)\} \pi_i \quad (15)$$

Here, $I(\cdot)$ is the indicator function, and $\alpha = 1 - g/(C'_{CM} - C'_{PM})$. $OB_n(\pi)$ in (3) can be reformulated likewise.

Next, let us compare $b_{OB}(\pi)$ with $b_{PM}(\pi)$. If $C'_{PM} < C_{OB} + \sum_i b(e_i)\pi_i$, PM is preferred to OB. As a result, if $R(\pi) < 1 - g/(C'_{CM} - C'_{PM})$, and $C'_{PM} < C_{OB} + \sum_i b(e_i)\pi_i$, the optimal policy is PM. Also, from the facts that $b_{OB}(\pi)$ is non-decreasing in $\prec_{st}$-ordering, and that $b_{PM}(\pi)$ is constant, we can derive the control limit for PM in closed form. Many previous maintenance studies based on a POMDP simply prove the "existence" of the control limit for PM. But for this problem, we analytically obtain the necessary and sufficient condition. Theorem 1 summarizes the results.

Theorem 1: Suppose that P is IFR, and upper-triangular. (a) For $\pi \prec_{st} \pi'(\pi)$, the region where the optimal policy is PM is defined by $\Omega_{PM} = \{\pi; R(\pi) < 1 - g/(C'_{CM} - C'_{PM}), C'_{PM} < C_{OB} + \sum b(e_i)\pi_i\}$, whereas PM cannot be optimal for $\pi \notin \Omega_{PM}$. (b) Furthermore, if $\delta^*(\pi) = PM$, $\delta^*(\hat{\pi}) = PM$ for $\pi \prec_{st} \hat{\pi}$.

This PM region in Theorem 1 defines the optimal PM region of the AM4R policy, as we will discuss in Section IV-D.

Corollary 1: Suppose that P is IFR, and upper-triangular. (a) If $R(\pi) < 1 - g/(C'_{CM} - C'_{PM})$, and $C'_{PM} \geq C_{OB} + \sum b(e_i)\pi_i$, $\delta^*(\pi) = OB$ for $\pi \prec_{st} \pi'(\pi)$. (b) If $R(\pi) \geq 1 - g/(C'_{CM} - C'_{PM})$, and $C'_{PM} < C_{OB} + \sum b(e_i)\pi_i$, $\delta^*(\pi) = NA$.

Finally, let us compare $b_{NA}(\pi)$ with $b_{OB}(\pi)$. We present the conditions under which NA is preferred to OB, and vice versa, in Lemma 3, and Lemma 4.

Lemma 3: If $R(\pi) \geq (C'_{CM} - C_{OB} - \sum b(e_i)\pi_i - g)/(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i^2)$, then $\delta^*(\pi) \neq OB$.

Similar to Lemma 2, Lemma 3 also explains that when the system is in a fairly good condition with a high reliability, we do not need to carry out costly inspection of the system. Along with Lemma 2, the following Corollary 2 specifies the sufficient condition for NA to be optimal; its proof follows directly from Lemma 2, and Lemma 3.

Corollary 2: If $R(\pi) \geq \max\{1 - g/(C'_{CM} - C'_{PM}), (C'_{CM} - C_{OB} - \sum b(e_i)\pi_i - g)/(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i^2)$, then $\delta^*(\pi) = NA$.

And, Lemma 4 specifies the sufficient condition under which OB is optimal.

Lemma 4: Suppose that $R(\pi) < (C'_{CM} - C_{OB} - \sum b(e_i)\pi_i - g)/(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i^2)$. If $\delta^*(\pi^2) = OB$, then $\delta^*(\pi) = OB$.

C. Structural Properties Along Sample Path

By extending the claim of Proposition 4, we can easily show that, when P is IFR, all of the states in the sample path emanating from any π is in increasing stochastic order as long as $\pi \prec_{st} \pi^2$. This allows us to apply all of the results developed in Section IV-B to the states along a sample path in increasing stochastic order. The following Corollary 3 summarizes them.

Corollary 3: Suppose that P is IFR, and upper-triangular. Then the states along a sample path satisfy the following properties for $\pi \prec_{st} \pi'(\pi)$.

(a) Any sample path is in $\prec_{st}$-increasing order. That is, $\pi \prec_{st} \pi^2 \prec_{st}, \cdots, \prec_{st} \Pi(\pi)$.
(b) $V_n(\pi)$, and $b(\pi)$ are non-decreasing along any sample path.
(c) Suppose that $R(\pi^q) \geq 1 - g/(C'_{CM} - C'_{PM})$. $\delta^*(\pi^k) \neq PM$ for $\forall k \leq q$. On the contrary, if $R(\pi^q) < 1 - g/(C'_{CM} - C'_{PM})$, $\delta^*(\pi^k) \neq NA$ for $\forall k \geq q$.
(d) There exists a critical number k^* such that $\delta^*(\pi^k) = PM, \forall k \geq k^*$, and $\delta^*(\pi^k) \neq PM$ otherwise. And, such k^* is given by $k^* = \max\{k1(\pi), k2(\pi)\}$ where

$$k1(\pi) = min\left\{k; R(\pi^k) < 1 - \frac{g}{C'_{CM} - C'_{PM}}\right\}, \quad (16)$$

$$k2(\pi) = min\left\{k; C_{OB} + \sum b(e_i)\pi_i^k > C'_{PM}\right\}. \quad (17)$$

D. The Monotonic At-Most-Four-Region Policy

Several previous studies establish the AM4R policy structure along an ordered subset of state space for POMDP problems in different maintenance settings. For example, Maillart [11] presents the AM4R structure along any straight line of $\prec_{lr}$-ordered information states when P is $TP2$ in her model.

In this section, we establish similar results for the presented problem under less stringent assumptions on the transition matrix, and information states. Specifically, we show that the optimal policy has the AM4R structure along a straight line of $\prec_{st}$-ordered states on IFR transition matrix. Consider two states π, and $\hat{\pi}$, for $\pi \prec_{st} \hat{\pi}$. Let us denote a state between π, and $\hat{\pi}$ by $\pi(\lambda) = \lambda\pi + (1-\lambda)\hat{\pi}$, $0 \leq \lambda \leq 1$. Here, higher λ implies a more deteriorated condition (we will show the reason in Theorem 2). Then, there exist at most three numbers λ_1, λ_2, λ_3 to divide the optimal policy regions as follows.

$$\delta^*(\pi(\lambda)) = min \begin{cases} NA, & \text{if } \lambda < \lambda_1 \text{ or } \lambda_2 < \lambda \leq \lambda_3 \\ OB, & \text{if } \lambda_1 \leq \lambda \leq \lambda_2 \\ PM, & \text{if} \lambda > \lambda_3 \end{cases} \quad (18)$$

That is, as λ increases, the optimal policy regions are divided into at most four regions with the order $NA \to OB \to NA \to PM$. To establish this AM4R structure, we first show the concavity of $V_n(\pi)$.

Lemma 5: $V_n(\pi)$ is piecewise linear concave for all n.

Now, we are ready to prove the monotonic AM4R structure along a $\prec_{st}$-increasing line.

Theorem 2: If P is IFR, the optimal policy has the monotonic AM4R structure along any straight line of information states in $\prec_{st}$-increasing order. Furthermore, the control limit to define optimal PM policy is defined by $\lambda^* = \inf\{\lambda; R(\pi(\lambda)) < 1 - g/(C'_{CM} - C'_{PM}), C'_{PM} < C_{OB} + \sum b(e_i)\pi(\lambda)_i\}$.

As Rosenfield [16] points out, the second NA region in the AM4R structure may seem counter-intuitive. In the following discussions, we establish the conditions under which we have

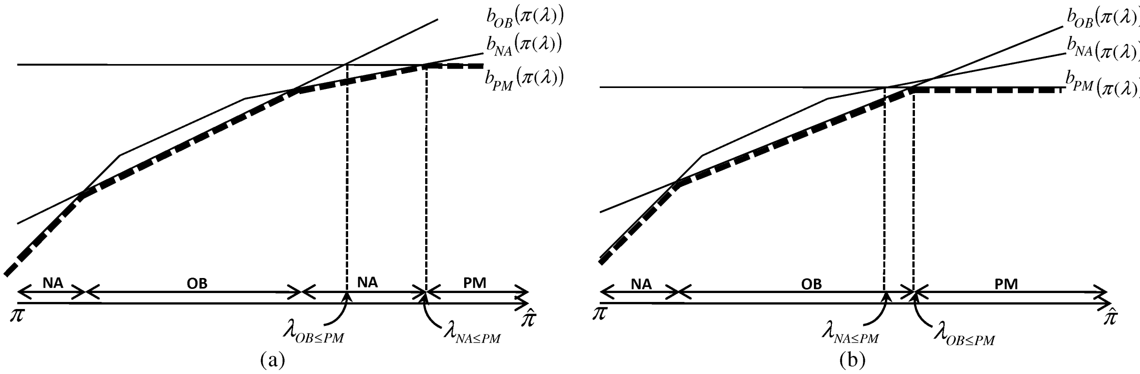

Fig. 2. The optimal policy structure: (a) monotonic AM4R structure; (b) monotonic AM3R structure.

the more intuitive AM3R policy structure. Let us define the critical numbers to divide the optimal policy regions as follows.

$$\lambda_{NA\le PM} = max\left\{\lambda; R(\pi(\lambda)) \ge 1 - \frac{g}{C'_{CM} - C'_{PM}}\right\} \quad (19)$$

$$\lambda_{OB\le PM} = max\left\{\lambda; C_{OB} + \sum b(e_i)\pi(\lambda)_i \le C'_{PM}\right\} \quad (20)$$

Note that, for $\lambda \le \lambda_{NA\le PM}$, NA is preferred to PM, and vice versa. Similarly, for $\lambda \le \lambda_{OB\le PM}$, OB is preferred to PM, and vice versa.

Corollary 4: If $\lambda_{NA\le PM} < \lambda_{OB\le PM}$, the optimal policy has the monotonic AM3R structure along any $\prec_{st}$-increasing straight line of information states with the order of $NA \rightarrow OB \rightarrow PM$. The optimal policy region for PM is given by $\{\pi(\lambda); C'_{PM} < C_{OB} + \sum b(e_i)\pi(\lambda)_i\}$.

Fig. 2 compares the two policy structures. Whether the optimal policy structure exhibits the AM4R or AM3R is highly dependent on the costs of PM. When the PM costs are relatively larger compared to the costs related to other actions, the structure is more likely to result in the AM4R structure, as shown in Fig. 2(a). Otherwise, when PM costs are comparable to other costs, the AM3R structure is more likely, as shown in Fig. 2(b).

In wind turbine operations, the repair costs (C'_{CM}) after an unplanned failure are considerably larger compared to the PM costs (C'_{PM}), as explained in Sections I and III. Also, in most cases, OB costs are not negligible because inspecting the physical condition by dispatching a crew is costly due to the high labor costs, and the long distance of wind farms from the operation centers [27]. This high cost implies that the presented optimal policy would more likely lead to the AM3R structure in wind turbine maintenance problems.

V. Algorithm

In Section III-B, we introduced the pure recursive technique to get the optimal policy. Now, using the structural policies developed so far, we can reduce the computational efforts substantially.

First, given the parameter values ($C_{CM}, C_{PM}, C_{OB}, W_{CM}$, W_{PM}, P, τ, and T), we obtain $b(e_i)$, $i = 1, \cdots, m+1$, and average cost g by applying policy (or value) iteration to the states on the extreme sample paths. Then, C'_{CM}, and C'_{PM} in (11), and (12) can be computed, respectively, as discussed in Section III-A. Then, we apply the following decision rules to attain the optimal policy for a given π.

- Suppose that there exists a π at which the optimal policy is PM. Then $\delta^*(\hat{\pi}) = PM$ for $\pi \prec_{st} \hat{\pi}, \forall\hat{\pi}$.
- Suppose that $R(\pi) < 1 - g/(C'_{CM} - C'_{PM})$. If $b_{OB}(\pi) > b_{PM}(\pi)$, then $\delta^*(\pi) = PM$. Otherwise, $\delta^*(\pi) = OB$.
- Suppose that $R(\pi) \ge 1 - g/(C'_{CM} - C'_{PM})$. Then $\delta^*(\pi) = NA$ if $b_{OB}(\pi) > b_{PM}(\pi)$, or if $R(\pi) \ge (C'_{CM} - C_{OB} - \sum b(e_i)\pi_i - g)/(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i^2)$.
- Suppose that $1 - g/(C'_{CM} - C'_{PM}) \le R(\pi) < (C'_{CM} - C_{OB} - \sum b(e_i)\pi_i - g)/(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i^2)$, and $b_{OB}(\pi) \le b_{PM}$. We apply the following recursive method, which improves the pure recursive technique.

— Step 1. Set $k = 1$;

— Step 2. If $R(\pi^k) < 1 - g/(C'_{CM} - C'_{PM})$, then $b(\pi^k) = \min\{b_{OB}(\pi^k), b_{PM}(\pi^k)\}$. Then apply the recursive set of equations (13) backward to get $b(\pi^{k-1}), \cdots, b(\pi)$. Otherwise, $k = k+1$, and go to Step 3.

— Step 3. If $\|\pi^{k+1} - \pi^k\| < \epsilon$, we apply (14) to get $b(\pi^k)$, and then step backwards along the path by comparing b_{NA} and b_{OB} to get $b(\pi^{k-1}), \cdots, b(\pi)$. Otherwise, $k = k+1$, and go back to Step 2.

The above method results in an optimal policy that can be analytically obtained from the closed-form expressions. We need to apply the recursive method only for the states whose reliabilities are between $1 - g/(C'_{CM} - C'_{PM})$, and $(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i - g)/(C'_{CM} - C_{OB} - \sum b(e_i)\pi_i^2)$. Even for the recursive method itself, as Step 2 shows, we do not have to proceed until we meet the stationary state $\Pi(\pi)$. Along the sample path, once we find the state whose optimal policy is not NA (that is, $R(\pi^k) \le 1 - g/(C'_{CM} - C'_{PM})$ for some π^k), we can compute $b(\pi^k)$ by comparing b_{PM} with $b_{OB}(\pi^k)$. Then we can step backwards by applying (13) until we get π. On the contrary, Step 3 occurs when the reliability at the stationary state is greater than $1 - g/(C'_{CM} - C'_{PM})$. In this case, PM cannot be optimal at all of the states along the sample path originating from π. Therefore, we only need to compare $b_{NA}(\pi^k)$ with $b_{OB}(\pi^k)$ when we step backwards to π.

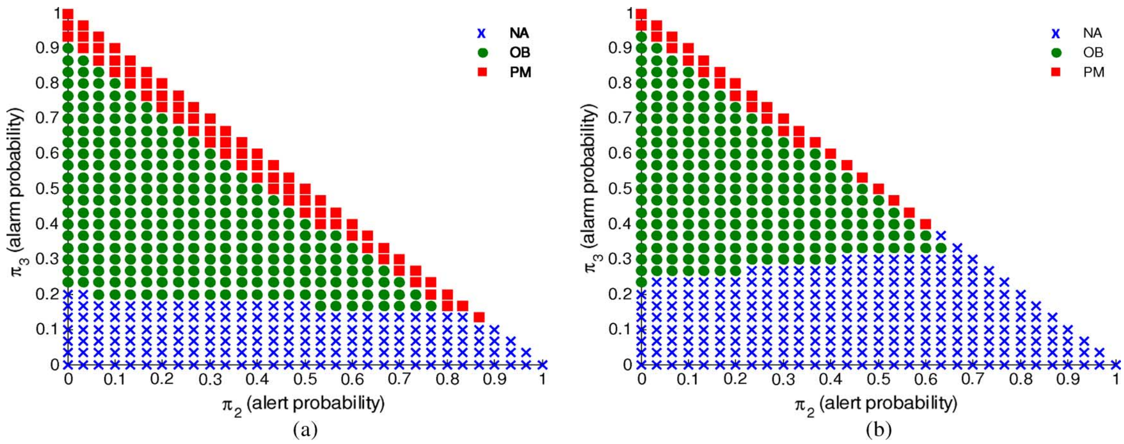

Fig. 3. Optimal policies. (a) $W_{PM} = 0.1, W_{CM} = 0.4$. (b) $W_{PM} = 0.4, W_{CM} = 0.4$

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VI. Numerical Examples

McMillan & Ault [6] show that the most critical failures in wind turbines are associated with the gearbox because of high capital cost, long lead time for repairs, difficulty in replacing a gearbox, and lengthy downtime compounded by adverse weather conditions. Therefore, we choose a gearbox among several components of a wind turbine to illustrate the presented methodology.

A. Example for Gearbox Maintenance

We choose appropriate parameter values based on published data, and discussions with our industry partners. For the costs to repair a gearbox, we refer to [27]. The total direct costs for CM, which include labor costs, crane rental, materials, and consumables, are $C_{CM} = 12,720$. The PM costs are about half that of the CM costs: $C_{PM} = 6,360$. For a 2.5 MW turbine, revenue loss during one week is $\tau = 8,820$. We set $C_{OB} = 1,000$, according to the suggestions of our industry partners. The monetary unit of each cost factor in this example is euros.

Typical downtime after failures may take from 600 hours (25 days) up to 60 days [6], [10], [28]. The major contribution of this lengthy down time is the long lead time when the spare parts and/or crew are not available. In this study, we assume that, upon failure, the lead time (T) for assembling repair crew and spare parts and travel time takes six weeks. We also assume that repairs can be carried out in about one week [6].

Generally, a transition matrix P can be generated from historical data by taking a long-run history about the deterioration states, and counting transitions. Due to the relatively short history of preventive maintenance practices in wind turbine industries, we do not yet have a transition matrix generated from an actual aging gearbox. So we use a P similar to the one used in the example in [11] with slight modifications. We will examine the sensitivity of P in the next subsection. We assume that the weekly-based deterioration process follows a Markovian behavior with the following IFR, upper-triangular P matrix.

$$P = \begin{bmatrix} 0.90 & 0.05 & 0.03 & 0.02 \\ 0.00 & 0.85 & 0.10 & 0.05 \\ 0.00 & 0.00 & 0.92 & 0.08 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{21}$$

Based on (1), we can represent the state of the gearbox as a four-dimensional row vector, $\pi = \{\pi_1, \pi_2, \pi_3, \pi_4\}$. The values π_1, π_2, and π_3 represent the probabilities of being in a *normal*, *alert*, and *alarm* state, respectively. The value π_4 represents the probability of being in a *failed* state.

Fig. 3 illustrates the optimal policies with two different stochastic weather environments. We can see that Ω_{OB}, and Ω_{PM} are convex sets. Also, if we draw a line between any two points, the policy regions are divided into at most three regions in most cases, which is consistent with the previous discussions that the AM3R structure might dominate over the AM4R structure in most real applications. Also notice that Ω_{PM} gets smaller as the chance of adverse weather conditions to prohibit PM increases. That is, with higher frequency of adverse weather conditions (that is, with higher W_{PM}), wind farm operators should be more conservative in carrying out PM because of possible production losses caused by interrupted or delayed jobs during harsh weather.

Fig. 4 superimposes the control limits developed in Section IV-B on the optimal policy for the same example in Fig. 3(a). Line 1 depicts the preference of NA to PM, or vice versa, with $R(\pi) = 1 - g/(C'_{CM} - C'_{PM})$. Line 2 is obtained from the comparison of b_{OB} and b_{PM} with $C'_{PM} = C_{OB} + \sum_i b(e_i)\pi_i$. Finally, Line 3 defines the area where NA is preferred to OB. The optimal policy of each area is as follows.

- PM in states above Line 1, and Line 2 (by Theorem 1).
- OB in states above Line 1, and below Line 2 (by Corollary 1(a)).
- NA in states below Line 1, and Line 3 (by Lemma 2, and Lemma 3).

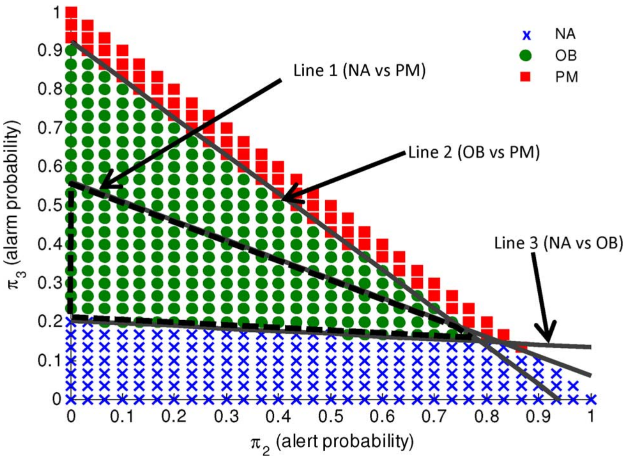

Fig. 4. Control limits superimposed on optimal policies for $W_{PM} = 0.1$, and $W_{CM} = 0.4$.

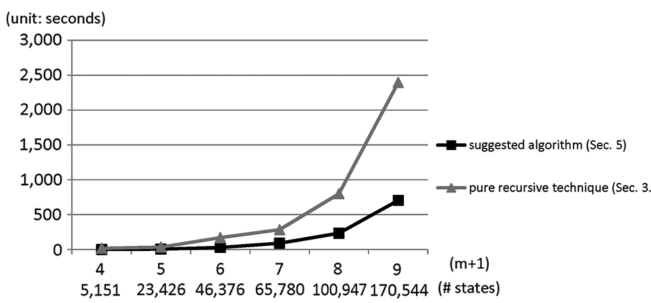

Fig. 5. Performance comparison where "$m+1$" denotes the dimension of states, and "# states" denotes the number of states that we evaluate to optimal policies.

- NA in states in the triangular area surrounded by Line 1, Line 2, and Line 3 (by Corollary 1(b)).

The only states whose optimal policy are not straightforward from these control limits are shown in the region surrounded by the dashed lines in Fig. 4. The optimal policy in this region is obtained by applying the improved recursive technique discussed in Section V.

B. Performance Comparison

Suppose that we want to find the optimal policy at every grid point, as shown in Figs. 3 and 4. As the dimension of states increases, computation time significantly increases when we use the pure recursive algorithm. Fig. 5 compares the performance of the suggested algorithm with the pure recursive technique. We use the same parameter values in the previous examples, and $W_{PM} = 0.1$ and $W_{CM} = 0.4$, but vary the transition matrix along a state size. The results indicate that the closed form of decision boundaries compounded by the improved recursive technique reduces the computation time by more than 70% over various sizes of the problem instances.

C. Sensitivity Analysis of Transition Matrix P

Considering difficulties to get a transition matrix P, we analyse the sensitivity of a transition matrix by applying four additional, different matrices, $P_i, i = 1, \cdots, 4$. P_1 represents a more slowly deteriorating system in a stochastic sense than P in (21). That is, each row vector of P_1 is stochastically less than the corresponding row vector of P. Let us denote this relationship by $P_1 \prec_{st} P$. Similarly, $P_2 \prec_{st} P$, and also $P_1 \prec_{st} P_2$. On the other hand, P_3, and P_4 represent more rapidly deteriorating systems than P, such that $P \prec_{st} P_3 \prec_{st} P_4$.

TABLE I
SENSITIVITY ANALYSIS ON P

P_k	ΔP_k	g	g_k	ΔG_k
P_1	10.3%	2549.0	2599.4	2.0%
P_2	6.1%	2549.0	2576.5	1.1%
P_3	5.7%	2549.0	2572.8	0.9%
P_4	10.1%	2549.0	2596.3	1.9%

We quantify the speeds of deterioration of a system with P_k, $k = 1, \cdots, 4$, compared to P, with the measure

$$\Delta P_k(\%) = \sum_{i=1}^{m} \sum_{j \geq i} \frac{|P(i,j) - P_k(i,j)|}{P(i,j)} \times 100 \tag{22}$$

where $P(i,j)$ is the element in the ith row and jth column of P matrix, and $P_k(i,j)$ is similarly defined. Note that the lower off-diagonal elements are not involved in (22) because we consider upper-triangular matrices. ΔP_k implies the relative difference of P_k, compared to P.

To measure the sensitivity of a transition matrix, we use simulation. Suppose that the actual system undergoes a deterioration process following a transition matrix P. We simulate the trajectories of system states following P from 136 different starting points. Here, 136 starting points are the points in the grid, similar to the grid points shown in Fig. 4. But to speed up the simulations, we use a coarser grid such that the distance between adjacent grid points is 2/3. From each starting point, the simulation is performed over 1,000 periods. At each period, we take actions as the optimal policy suggests. Then the costs are averaged. This process is repeated 30 times. That is, we gain the average cost g by the simulations on 136 different starting points $\times$ 1,000 periods $\times$ 30 trajectories (runs).

However, suppose that we do not know the transition matrix exactly, so we incorrectly use the transition matrix P_k to attain optimal policies, while the actual deterioration process follows P. We apply the similar simulation process, but we use P_k to decide the optimal policy. Then, we compute the average cost g_k. From the results of the simulations, we quantitatively measure the sensitivity of each transition matrix by

$$\Delta G_k = \frac{g_k - g}{g} \times 100. \tag{23}$$

Table I summarizes the results. The fourth column (that is, g_k) shows that the average costs increase as the assumed transition matrix P_k deviates from the actual transition matrix P. However, the difference is not significant, as the fifth column (that is, ΔG_k) indicates. Even when the values of the actual transition matrix deviates from the assumed transition matrix values by about 10% such as P_1 and P_4, the increased cost is about 2.0%

on average. When the element values are different by 5–6% such as P_2 and P_3, average costs are increased by around 1%.

Although the results show that the average costs are not seriously affected by the deviation of the assumed transition matrix from the actual one, we recommend making considerable efforts to accumulate data regarding system deterioration. Rademakers *et al.* [27] also suggest that industry parties should share data for the improvement of O&M for wind turbines. For conventional power systems, these data for critical equipment such as circuit breakers and transformers have been accumulated, and several preventive maintenance strategies have been introduced based on historical data [29]–[31]. Similar efforts are necessary in wind power industries.

VII. Summary

Despite the vast potential capacity of wind power, the share of wind energy still remains a small portion of the entire energy market. One of the critical factors for enhancing marketability of wind energy is reducing O&M costs. This factor exists because dispatching maintenance crews with heavy-duty equipment to remote wind farm sites is very expensive. O&M for wind turbines has unique aspects that call for a new maintenance model, and further analysis. Wind turbines are typically located in remote areas, and operate under irregular, non-stationary conditions. So the suitability of executing or continuing a repair job depends on stochastic weather conditions. However, most maintenance studies in the literature consider static environmental conditions. Also, lead time for repairs is typically longer than for other equipment, and revenue losses upon failures are significant.

In this study, we use probabilistic cost modeling to quantify risks and uncertainties, and develop an O&M decision model for wind turbines that incorporates these practical aspects. The model has potential to provide practical operations and maintenance guidelines, to reduce repair costs, and increase marketability of wind energy. We also provide an analysis that gives insights into the model structure, and enables efficient numerical solutions. We analytically derive a set of closed-form expressions for the optimal policy, and show how these results can be utilized to solve large problems. We extend the AM4R structure under weaker assumptions than a previous study in the literature, and demonstrate the conditions under which this AM4R structure becomes the AM3R structure. We believe that these results can be applicable to other general aging systems.

As future work, we plan to extend the model to incorporate multiple wind turbines. In this study, we assume independence of each turbine operation. However, when turbines are operating, the rotating blades change wind speeds, and can affect the operation of other wind turbines, a condition known as "wake effects". It would be interesting to see how robust the recommended maintenance policy can perform in a wind farm that houses many wind turbines. As a part of our ongoing research, we have been developing a large-scale simulation model for wind farm operations using the discrete event specification (DEVS) formalism [32] with hundreds of wind turbines [33]. We plan to integrate the DEVS simulation model with the proposed O&M model to validate optimal policies, and to see if further modifications are necessary when we consider multiple turbines.

Appendix

To save space, we only provide the proofs of Lemma 2, Theorem 1, Lemma 5, and Theorem 2 which we believe are the most important claims in our theoretical findings. The proofs of other claims are also available from the corresponding author's website [34].

Proof of Lemma 2:

$$\begin{aligned} b_{NA}(\pi) - b_{PM}(\pi) &= C'_{CM}\,(1 - R(\pi)) + b(\pi^2)R(\pi) - g - C'_{PM} \quad (24)\\ &= (C'_{CM} - C'_{PM})(1 - R(\pi)) \\ &\quad - g + \left(b(\pi^2) - C'_{PM}\right) R(\pi) \quad (25)\end{aligned}$$

Note that $b(\pi^2) \le C'_{PM}$. Consequently, if $(C'_{CM} - C'_{PM})(1 - R(\pi)) - g \le 0$ (or equivalently, $R(\pi) \ge 1 - g/(C'_{CM} - C'_{PM})$), NA is preferred to PM.

Next, Consider the case that $(C'_{CM} - C'_{PM})(1 - R(\pi)) - g > 0$. Let us assume that $\delta^*(\pi) = NA$. Then,

$$\begin{aligned} b(\pi^2) - b(\pi) &= b(\pi^2) - \left(C'_{CM}\,(1 - R(\pi)) + b(\pi^2)R(\pi) - g\right) \quad (26)\\ &= \left(b(\pi^2) - C'_{PM}\right)(1 - R(\pi)) \\ &\quad - (C'_{CM} - C'_{PM})(1 - R(\pi)) + g. \quad (27)\end{aligned}$$

Equation (26) holds from the assumption $\delta^*(\pi) = NA$; and thus, $b(\pi) = C'_{CM}(1 - R(\pi)) + b(\pi^2)R(\pi) - g$. Note that in (27), $b(\pi^2) \le C'_{PM}$. Therefore, when $(C'_{CM} - C'_{PM})(1 - R(\pi)) - g > 0$, $b(\pi^2) \le b(\pi)$ with the assumption of $\delta^*(\pi) = NA$. But this result contradicts that $b(\pi^2) \ge b(\pi)$ for $\pi \prec_{st} \pi'(\pi)$ from Lemma 1. Therefore, when $(C'_{CM} - C'_{PM})(1 - R(\pi)) - g > 0$, or equivalently, $R(\pi) < 1 - g/(C'_{CM} - C'_{PM})$, NA cannot be optimal. ■

Proof of Theorem 1: The first part is straightforward from Lemma 2, and the above discussions. Regarding the second part, NA cannot be optimal at $\hat{\pi}$ from the fact that $R(\hat{\pi}) \le R(\pi)$ for $\pi \prec_{st} \hat{\pi}$. Also, because $b(e_i)$ is non-decreasing in i, $\sum_i b(e_i)\pi_i$ is also non-decreasing in $\prec_{st}$-ordering from Proposition 2, and so is $b_{OB}(\pi)$. This result leads to $b_{OB}(\hat{\pi}) \ge b_{OB}(\pi)$. But, $b_{PM}(\pi)$ is constant. Thus, when $\delta^*(\pi) = PM$, OB cannot be optimal at $\hat{\pi}$ as well, which concludes the second part of the Theorem. ■

Proof of Lemma 5: We apply the similar induction technique used in [11]. Suppose that $CM_0(e_{m+1}) = C_{CM}$. Also, suppose that $V_0(\pi) = 0$ for $\forall\pi$ for an operating system. $NA_1(\pi) = C_{CM}(1 - R(\pi))$ is linear in π. $OB_n(\pi)$ is a hyperplane of π, and $PM_n(\pi)$ is constant in π $\forall n$. Therefore, $V_1(\pi)$ is piecewise linear concave because the minimum of linear functions is piecewise linear concave. Now, suppose that $V_n(\pi)$ is piecewise linear concave such that $V_n(\pi) = min\{\pi \cdot a_n^T; a_n \in A_n\}$ where a_n is a $1 \times (m + 1)$ dimensional column vector. We only need to examine $NA_{n+1}(\pi)$ to show the piecewise linear concavity of $V_{n+1}(\pi)$. The first term of $NA_{n+1}(\pi)$, (that is,

$(\tau T + CM_{n-T-1}(e_m + 1))(1 - R(\pi)))$ is linear in π. The second term of $NA_{n+1}(\pi)$ is

$$R(\pi)V_n(\pi^2) = R(\pi)min\left\{\pi^2 \cdot a_n^T a_n \in A_n\right\} \quad (28)$$

$$= R(\pi)min\left\{\left[\frac{(\pi P)_1}{R(\pi)}, \frac{(\pi P)_2}{R(\pi)}, \cdots, \frac{(\pi P)_m}{R(\pi)}, 0\right] \cdot a_n^T; a_n \in A_n\right\} \quad (29)$$

$$= min\left\{[(\pi P)_1, (\pi P)_2, \cdots, (\pi P)_m, 0] \cdot a_n^T; a_n \in A_n\right\} \quad (30)$$

$$= min\left\{\pi \cdot a_{n+1}^T; a_{n+1} \in A_{n+1}\right\} \quad (31)$$

Because $R(\pi)V_n(\pi^2)$ is the minimum of hyperplanes, it is piecewise linear concave, which makes $NA_{n+1}(\pi)$ also piecewise linear concave. Consequently, $V_{n+1}(\pi)$ is piecewise linear concave. And the claim holds $\forall n$ by induction. ■

Proof of Theorem 2: Consider the two states $\pi(\lambda_1)$, and $\pi(\lambda_2)$ between π, and $\hat{\pi}$ ($\pi \prec_{st} \hat{\pi}$) where $\pi(\lambda_j) = \lambda_j\pi + (1-\lambda_j)\hat{\pi}$, for $j = 1, 2$, and $0 \leq \lambda_1 \leq \lambda_2 \leq 1$. Then, from $\sum_{i\geq j}\pi_i \prec_{st} \lambda_1\sum_{i\geq j}\pi_i + (1-\lambda_1)\sum_{i\geq j}\hat{\pi}_i \prec_{st} \sum_{i\geq j}\hat{\pi}_i$, we have $\pi \prec_{st} \pi(\lambda_1) \prec_{st} \hat{\pi}$. In a similar way, we can easily show that $\pi(\lambda_1) \prec_{st} \pi(\lambda_2) \prec_{st} \hat{\pi}$. Therefore, $\pi(\lambda)$ is $\prec_{st}$-increasing in λ, which implies that $b_{NA}(\pi(\lambda))$, and $b_{OB}(\pi(\lambda))$ is non-decreasing in λ. But, $b_{PM}(\pi(\lambda))$ is constant. Hence, there exists a control limit λ^* such that for any $\lambda > \lambda^*$, PM is optimal. The value of λ^* is straightforward from Theorem 1. Next, let us consider $0 \leq \lambda \leq \lambda^*$. For this region, we already know that PM cannot be optimal from Theorem 1. In Lemma 5, we show that $NA_n(\pi)$ is piecewise linear concave. Thus $b_{NA}(\pi)$ is also piecewise linear concave, but $b_{OB}(\pi)$ is a hyperplane. Thus, $\{\pi; b_{NA}(\pi) \geq b_{OB}(\pi)\}$ is a convex set, and thus, $\{\lambda; b_{NA}(\pi(\lambda)) \geq b_{OB}(\pi(\lambda)), 0 \leq \lambda \leq \lambda^*\}$ is also a convex set. This concludes the AM4R structure. ■

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